Optimization Model for Prediction of Compressive Strength of Concrete Containing Fly Ash and Quarry Dust Based on Scheffe's Simplex Theory

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Abstract: **This work involves the application of Scheffe's optimization technique toobtain the mathematical model of response function for the optimization of compressive strength of a six- componentconcrete madewithcement, fly ash, sand, quarry dust, granite and water.A total of 84 standard 150x150x150mm cubes was cast, fromforty-two (42) mix ratios.The first twenty-one (21) were used to determine the coefficients of the model, while the remainingtwenty-one were used to validate the model.The MATLAB software developed gave optimum compressive strength of 43.01N/mm^{2}after 28 days wet curing for a mix ratio of 0.91:0.094:0.868:0.138:2.18:0.40 (i.e. cement: fly ash:sand:quarry dust: granite: water).The output of the mathematical models compared favourably with the corresponding experimental results and the predictions from the response function were tested for adequacy using the statistical Fischer test and found to be adequate at 95% confidence level. The model derived in this study can be used to predict mix ratios for any desired compressivestrength ofa six-component concrete system within a 6,2 factor space.**

Keywords: **Compressive Strength, Quarry Dust, Fly Ash, Mathematical model, Scheffe's, Optimization, Concrete.**

1. INTRODUCTION

The problem of affordable housing and construction economy continue to remain a topic of discussion for researchers. Concrete is one of the most popular construction materials worldwide and itsuse has witnessed tremendous increase over the years.Concrete is a mixture of cement, fine aggregate and coarse aggregate and water and when sufficiently hardened is used in supporting various structural loads. The cost of production of concrete heavily impacts on construction cost. Most of the engineering characteristics of concrete depend on proportions of the constituent materials. The river sand which is commonly used as fine aggregate in the production of concrete, poses the problem of acute shortage in some areas, resulting inadditional cost of transportation from far-way sources. This emphasizes the need to optimize the use of river sand with a view to reducing the cost of concrete production.

2. AIMS AND OBJECTIVE OF STUDY

Efforts have been made by various researchers to optimize the benefits of quarry dust and fly ash as alternative replacements for sand and cement in concrete production.

Quarry dust is a by-product from the crushing process during production of granite aggregates.Quarry dust is known to increase the strength of concrete over concrete made with equal quantities of river sand. Fly ash is a pozzolanic material, obtained as by-product of burned coal from thermal power plants.Several studies have been conducted on the strength of concrete, containing quarry rock dust and fly ash. Ephraimand Rowland-Lato [1] concluded that total replacement of conventional granite and natural sand with 10mm washed gravel aggregate and quarry rock dust respectively is possible without compromising the compressive strength of concrete. Jamale and Kawade [2] studied the effect ofpartial replacement of cement with fly ash and sand with quarry duston the strength properties of grade 40 concrete.The authors confirmed that the optimal compressive strength occurred with 5% fly ash and 15% quarry rock dust replacement.These researches confirm that quarry dust and fly ash can be utilized in concrete for partial replacement of sand and cement. Anyaogu and Ezeh [3]studied optimization of compressive strength of concrete, containing fly ash blended cement and normal aggregates, using Scheffe's simplex theory. The research concluded that the model derived can be used to predict mix ratios for any desired strength of fly ash blended cement concrete within the (5,2) factor space of asimplex model.

The aim of this study is to apply Scheffe's optimization model to a six-component concrete mix and obtain mathematical equation for the optimization of a given strength characteristic of the concrete. The objective included testing concrete from various mix ratios where cement and sand are partially replaced with fly ash and quarry dust and developing mathematical model that can be used to predict the compressive strength of concrete, based on the generated experimental data.

3. MATERIALS AND METHODS

The materials used in the study included cement, sand, granite chippings, quarry dust, fly ash and water. The methods encompassed the procedures for concrete mix design and development of Simplex model and its validation.

3.1 Materials

i. Ordinary Portland cement of Dangote Brand that conforms to BS 12 [4] was used as the binder in the concrete mixes investigated.

ii. The water used was potable and obtained from the University water mains.

iii. Coarse aggregate and quarry dust were obtained from Crushed Rock Industries quarry in Ishiagu, along Enugu-Port Harcourt Express Way, Ebonyi State, Nigeria. The granite had a maximum size of 20mm and washed free from excessive dust and debris.

iv. River sand was obtained from Choba in RiversState, Nigeria. The fine aggregates were sharp and free from clay and debris. The grading of the aggregates was carried out to BS 812: 103: Part 1, 1975.

v. Fly ash, otherwise known as pulverized fuel ash (PFA) is a pozzolanic material. Fly ash used as a partial replacement for cement in various mix proportions was obtained from the thermal coal station at Oji River, Enugu State, Nigeria.

3.2 Concrete Mix Design

The concrete system, containing cement, fly ash, sand, quarry dust and granite rock chippings, was designed in accordance with the Department of environment DoE procedure for a target strength of 40 N/mm^2. The detailed calculations are presented in Appendix 1. The mix proportions of cement: fly ash: sand: quarry dust: granite: water were obtained as 0.85:0.15:0.75:0.25:2.0:0.40.

3.3 Simplex Lattice Design Formulation for (6,2) System

Scheffe's model can be adapted to represent a six-component concrete mix containing cement, fly ash, sand, quarry dust, granite and water, by the hexahedron simplex matrix shown in Fig.1The modal coordinates are the pseudo components of the matrix(Scheffe [5]).

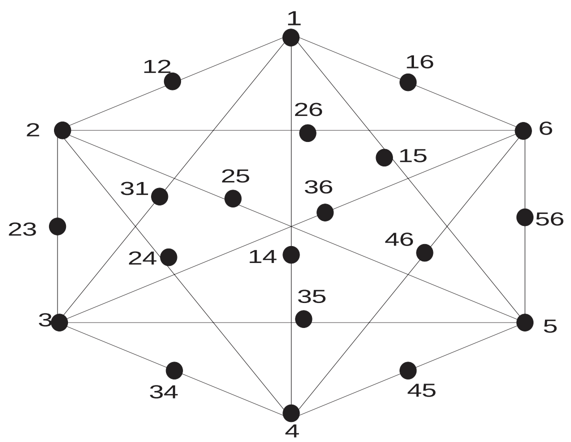

Fig 1: A (6,2) Hexahedron Simplex Lattice, representing Six-Component Concrete Mix

The relation between the actual components and the pseudo components is according to Osadebe and Ibearuegbulem[6]defined by the following equation:

$$\{S\}= [A]\{X\} \tag{1}$$

where S, A and X, represent the actual mix ratios, coefficient of relation matrix, and pseudo mix ratios respectively.

S and X are six component vectorsand Ais 6x6 matrix of coefficients. The value of matrix A was obtained from the first six mix ratios comprising the designed and modified mix ratios.

In order to satisfy the requirement of a 6.2 Scheffe's model, the following six mix ratios of cement: fly ash:sand: quarry dust:granite: water were generated from the basix mix designed in 3.2:

A_1[0.41,0.95,0.05,0.90,0.10,2.32],A_2[0.38,0.90,0.10,0.87,0.15,1.98],

A_3[0.39,0.88,0.12,0.80,0.20,2.11],A_4[0.40,0.85,0.15,0.75,0.25,2.00],

A_5[0.40,0.80,0.20,0.70,0.30,2.22],A_6[0.42,0.75,0.25,0.60,0.50,2.33].

The corresponding pseudo components from Fig.1 are: X_1[1,0,0,0,0,0],X_2[0,1,0,0,0,0],X_3[0,0,1,0,0,0],X_4[0,0,0,1,0,0],X_5[0,0,0,0,1,0],X_6[0,0,0,0,0,1],

Substituting X_i and S_i into Equation 1 andtransposing thevalues of A matrixwere obtained as:

$$[A] = \begin{pmatrix} 0.41 & 0.38 & 0.39 & 0.40 & 0.40 & 0.42 \\ 0.95 & 0.90 & 0.88 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.12 & 0.15 & 0.20 & 0.2 \\ 0.90 & 0.87 & 0.80 & 0.75 & 0.70 & 0.60 \\ 0.10 & 0.15 & 0.20 & 0.25 & 0.30 & 0.50 \\ 2.32 & 1.98 & 2.11 & 2.00 & 2.22 & 2.33 \end{pmatrix} \tag{2}$$

The first six components arelocated at the vertices of thehexahedronfactor space of the simplex lattice in Fig.1. Fifteen other pseudo components, located at mid-point of the lines joining the vertices of the simplex lattice, are: X_{12}[$\frac{1}{2}$,$\frac{1}{2}$,0,0,0,0],X_{13}[$\frac{1}{2}$,0,$\frac{1}{2}$,0,0,0],X_{14}[$\frac{1}{2}$,0,0,$\frac{1}{2}$,0,0],X_{15}[$\frac{1}{2}$, 0,0,0,$\frac{1}{2}$,0],X_{16}[$\frac{1}{2}$,0, 0,0,0,$\frac{1}{2}$],X_{23}[0,$\frac{1}{2}$,$\frac{1}{2}$, 0,0,0],X_{24}[0,$\frac{1}{2}$,0,$\frac{1}{2}$,0,0],X_{25}[$\frac{1}{2}$,0, 0,0, $\frac{1}{2}$,0],X_{26}[$\frac{1}{2}$,0,0, 0,0,$\frac{1}{2}$],X_{34}[0,0,$\frac{1}{2}$,$\frac{1}{2}$,0,0],X_{35}[0,0,$\frac{1}{2}$,0 ,$\frac{1}{2}$,0],X_{36}[0,0,$\frac{1}{2}$, 0,0,$\frac{1}{2}$],X_{45}[0, 0,0,$\frac{1}{2}$,$\frac{1}{2}$,0],X_{46}[0,0, 0,$\frac{1}{2}$,0,$\frac{1}{2}$],X_{56}[0,0,0, 0,$\frac{1}{2}$,$\frac{1}{2}$].

3.4 Formulation of Optimization Model based on Scheffe's Simplex Theory

According to Scheffe[5], for a six-component mixture, such as concrete containing fly ash and quarry dust, the proportion X_i of the i^{th} component of the mixture must satisfy the following inequality.

$$0 \leq X_i \leq 1 \tag{3}$$

Also, the sum of all proportions or constituents of any concrete mix must be equal to unity i.e

$$\sum_{i=1}^{q} X_i = 1 \tag{4}$$

Thus, for the six-component concrete being considered in this study,

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 1 \tag{5}$$

The general equation for regression is expressed as:

$$Y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijkl} X_i X_j X_k X_l + \sum b_{ijklm} X_i X_j X_k X_l X_m + e \tag{6}$$

where

b_i, b_{ij}, and b_{ijk} are constants. x_i, x_j and x_k are pseudo components, Y representsthe response and e is the error.

The number of coefficients K, of the polynomial is determined by the equation

$$K= C_n^{q+n-1} = {(q+n-1)!}/{(q-1)! * (n)!} \tag{7}$$

where

q is the number of components in the mixture, and n is the degree of the polynomial.

For the six-component mixture with two degrees polynomial, the number of coefficients K is twenty-one.

Expanding (6) by substituting the values of i and j (0≤I,j ≤6)will give:

$$Y = b_o + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{16}X_1X_6 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{26}X_2X_6 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{36}X_3X_6 + b_{45}X_4X_5 + b_{46}X_4X_6 + b_{56}X_5X_6 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 + b_{55}X_5^2 + b_{66}X_6^2 + e \quad (8)$$

To obtain the coefficients, first multiply Equation 5 by bo. This gives

$$b_o = b_oX_1 + b_oX_2 + b_oX_3 + b_oX_4 + b_oX_5 + b_oX_6 (9)$$

Then multiply same Equation (5) successively by X_1, X_2, X_3, X_4, X_5, X_6. After rearranging the products, we obtain the following expressions

$$\left.\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5 - X_1X_6 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5 - X_2X_6 \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5 - X_3X_6 \\ X_4^2 &= X_4 - X_1X - X_2X_4 - X_3X_4 - X_4X_5 - X_4X_6 \\ X_5^2 &= X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5 - X_5X_6 \\ X_6^2 &= X_6 - X_1X_6 - X_2X_6 - X_3X_6 - X_4X_6 - X_5X_6 \end{aligned}\right\} \quad (10)$$

Now, substituting Equations (9) and (10) into Equation (8),re-arranging and introducing the constant β, we have

$$Y = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{16}X_1X_6 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{26}X_2X_6 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{36}X_3X_6 + \beta_{45}X_4X_5 + \beta_{46}X_4X_6 + + \beta_{56}X_5X_6 + e \quad (11)$$

where

$\beta_i$andX_i are the coefficients of the response equation and pseudo components of the mix respectively.The coefficients β_i and β_{ij} are defined as follows:

$$\left.\begin{aligned} \beta_i &= b_o + b_i + b_{ii} \\ \beta_{ij} &= b_{ij} - b_{ii} - b_{jj} \end{aligned}\right] \quad (12)$$

Equation (11) can be reduced furthertoa more concise form:

$$Y = \sum_{i=1}^{6} \beta_i X_i + \sum_{i=1}^{6} \beta_j X_j + \sum_{i \le j \le 6}^{6} \beta_{ij} X_i X_j \quad (13)$$

Equation (13) is the combined response to the pure component, i and j and the binary component ij. Denoting the response function for the pure component, i and j and that of binary components, ij as $Y_i$and $Y_j$and Y_{ij} respectively, then

$$Y_i = \sum_{i=1}^{6} \beta_i X_i \quad (14)$$

$$Y_j = \sum_{j=1}^{6} \beta_j X_j \quad (15)$$

$$Y_{ij} = \sum_{i=1}^{6} \beta_i X_i + \sum_{i \le j \le 6}^{6} \beta_{ij} X_i X_j \quad (16)$$

If the response at i^{th} point on the factor space is Y_i, then at point 1, the pseudo component $X_1 = 1$ andX_2, X_3, X_4, X_5, X_6 are all equal to zero. Substituting these values into Equation (14), gives

$$Y_i = \beta_i (17)$$

Similarly, for j^{th} point $Y_j = \beta_j$

$$Y_j = \beta_j (18)$$

For intermediate point 12, that is the mid-point of the line connecting points 1 and 2 of the factor space, the components $X_1 = \frac{1}{2}$; $X_2 = \frac{1}{2}$ and $X_3, X_4, X_5 = 0$.

Substituting these values into Equation (16), the response, Y_{12} becomes

$$Y_{12} = \frac{1}{2}\beta_1 + \frac{1}{2}\beta_2 + \frac{1}{4}\beta_{12}$$

Similarly

$$\left.\begin{aligned} Y_{13} &= \tfrac{1}{2}\beta_1 + \tfrac{1}{2}\beta_3 + \tfrac{1}{4}\beta_{13} \\ Y_{14} &= \frac{1}{2}\beta_1 + \frac{1}{2}\beta_4 + \frac{1}{4}\beta_{14} \end{aligned}\right\} \quad (19)$$

Equations (19) can be expressed in the form.

$$Y_{ij} = \tfrac{1}{2}\beta_i + \tfrac{1}{2}\beta_j + \tfrac{1}{4}\beta_{ij} \quad (20)$$

Multiplying Equation (20) by 4 andre-arranging,wehave

$$\beta_{ij} = 4Y_{ij} \text{-} 2Y_i \text{-} 2Y_j (21)$$

Finally, using Equation (18) and (21) in Equation (11) we have

$$Y = Y_1X_1(2X_1 - 1) + Y_2X_2(2X_2 - 1) + Y_3X_3(2X_3 - 1) + Y_4X_4(2X_4 - 1)$$

$$+ Y_5X_5(2X_5 - 1) + Y_6X_6(2X_6 - 1) + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 +$$

$$4Y_{15}X_1X_5 + 4Y_{16}X_1X_6 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{25}X_2X_5 + 4Y_{26}X_2X_6$$

$$+ 4Y_{34}X_3X_4 + 4Y_{35}X_3X_5 + 4Y_{36}X_3X_6 + 4Y_{45}X_4X_5 + 4Y_{46}X_4X_6 + 4Y_{56}X_5X_6 (22)$$

In Equation (22) the Y_1, Y_2……….. Y_{56} are experimental compressive strength values to be determined from appropriate laboratory tests.

3.5 Concrete Mix Ratios

The real and pseudo mix ratios are defined as follows: 6 at the vertices and 15 at the intermediate points of lines joining the vertices. The complete 21 mixes are obtained by multiplying sequentially the basic mixes of Equation 2 by the corresponding pseudo values at the vertices and the midpoints. The actual and the corresponding pseudo ratios are shown in Table 1.

TABLE 1: ACTUAL AND PSEUDO MIX RATIOS OF THE MODEL

Points	Actual Mix ratios						Pseudo Mix ratios					
	Water	Cement	Fly Ash	Sand	Quarry dust	Granite	Water	Cement	Fly Ash	Sand	Quarry dust	Granite
	S_1	S_2	S_3	S_4	S_5	S_6	X_1	X_2	X_3	X_4	X_5	X_6
Y_1	0.41	0.95	0.05	0.90	0.10	2.32	1	0	0	0	0	0
Y_2	0.38	0.90	0.10	0.87	0.15	1.98	0	1	0	0	0	0
Y_3	0.39	0.88	0.12	0.80	0.20	2.11	0	0	1	0	0	0
Y_4	0.40	0.85	0.15	0.75	0.25	2.00	0	0	0	1	0	0
Y_5	0.40	0.80	0.20	0.70	0.30	2.22	0	0	0	0	1	0

Y_6	0.42	0.75	0.25	0.60	0.50	2.33	0	0	0	0	0	1
Y_{12}	0.395	0.93	0.075	0.885	0.13	2.15	0.5	0.5	0	0	0	0
Y_{13}	0.40	0.915	0.085	0.85	0.15	2.22	0.5	0	0.5	0	0	0
Y_{14}	0.405	0.90	0.10	0.825	0.175	2.16	0.5	0	0	0.5	0	0
Y_{15}	0.405	0.875	0.125	0.80	0.20	2.27	0.5	0	0	0	0.5	0
Y_{16}	0.415	0.85	0.15	0.85	0.25	2.33	0.5	0	0	0	0	0.5
Y_{23}	0.385	0.89	0.11	0.835	0.175	2.05	0	0.5	0.5	0	0	0
Y_{24}	0.39	0.875	0.13	0.81	0.20	1.99	0	0.5	0	0.5	0	0
Y_{25}	0.39	0.85	0.15	0.785	0.23	2.10	0	0.5	0	0	0.5	0
Y_{26}	0.40	0.825	0.175	0.785	0.275	2.16	0	0.5	0	0	0	0.5
Y_{34}	0.395	0.865	0.135	0.78	0.23	2.10	0	0	0.5	0.5	0	0
Y_{35}	0.395	0.84	0.16	0.75	0.25	2.22	0	0	0.5	0	0.5	0
Y_{36}	0.405	0.815	0.185	0.75	0.30	2.48	0	0	0.5	0	0	5
Y_{45}	0.40	0.825	0.175	0.725	0.28	2.11	0	0	0	0.5	0.5	0
Y_{46}	0.41	0.80	0.20	0.725	0.33	2.17	0	0	0	0.5	0	0.5
Y_{56}	0.41	0.775	0.225	0.70	0.35	2.28	0	0	0	0	0.5	0.5

In order to validate the model, extra twenty-one points (C_1, C_2, C_3, C_4, C_5, C_6, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{23}, C_{24}, C_{25}, C_{26}, C_{34}, C_{35}, C_{36}, C_{45}, C_{46}, C_{56}) of observation were used. These observations provided control mix ratios needed for the concrete mixes for this study. The mix ratios (actual and pseudo) for the work are shown in Table3, while those of the forty-two mix ratios (comprising 21 mix ratios for the trial mixes and 21 for control mixes) are shown in Table 2.

TABLE 2: ACTUAL AND PSEUDO COMPONENTS OF CONTROL OBSERVATION POINTS

Points	Actual Mix ratios						Pseudo Mix ratios					
	Water	Cement	Fly Ash	Sand	Quarry dust	Granite	Water	Cement	Fly Ash	Sand	Quarry dust	Granite
	S_1	S_2	S_3	S_4	S_5	S_6	X_1	X_2	X_3	X_4	X_5	X_6
C_1	0.393	0.909	0.09	0.856	0.15	2.135	0.3333	0.3333	0.3333	0	0	0
C_2	0.40	0.893	0.107	0.816	0.185	2.14	0.3333	0	0.3333	0.3333	0	0
C_3	0.403	0.866	0.133	0.783	0.217	2.18	0.3333	0	0	0.3333	0.3333	0
C_4	0.41	0.833	0.167	0.733	0.30	2.29	0.3333	0	0	0	0.3333	0.3333
C_5	0.395	0.895	0.105	0.83	0.175	2.10	0.25	0.25	0.25	0.25	0	0
C_6	0.385	0.858	0.142	0.768	0.263	2.19	0.25	0	0.25	0.25	0	0.25
C_{12}	0.395	0.883	0.117	0.818	0.188	2.16	0.25	0.25	0.25	0	0.25	0
C_{13}	0.403	0.820	0.180	0.713	0.313	2.17	0	0	0.25	0.25	0.25	0.25
C_{14}	0.403	0.863	0.137	0.78	0.25	2.16	0.25	0	0.25	0.25	0	0.25
C_{15}	0.398	0.906	0.094	0.868	0.138	2.183	0.5	0.25	0.25	0	0	0
C_{16}	0.41	0.833	0.167	0.725	0.325	2.273	0.25	0	0.25	0	0	0.5
C_{23}	0.398	0.906	0.094	0.844	0.160	2.15	0.40	0.20	0.20	0.20	0	0
C_{24}	0.40	0.866	0.134	0.784	0.24	2.15	0.20	0.20	0.20	0.20	0	0.20
C_{25}	0.404	0.846	0.154	0.750	0.27	2.20	0.20	0	0.20	0.20	0.20	0.20
C_{26}	0.402	0.850	0.150	0.764	0.260	2.17	0.20	0.20	0	0.20	0.20	0.20
C_{34}	0.398	0.836	0.164	0.744	0.280	2.10	0	0.20	0.20	0.20	0.20	0.20
C_{35}	0.40	0.866	0.134	0.784	0.24	2.15	0.20	0.20	0.20	0.20	0	0.20
C_{36}	0.401	0.841	0.16	0.747	0.28	2.16	0.10	0.10	0.20	0.20	0.20	0.20
C_{45}	0.402	0.868	0.132	0.787	0.225	2.18	0.30	0.10	0.10	0.20	0.20	0.10
C_{46}	0.401	0.848	0.152	0.764	0.26	2.20	0.20	0.10	0.10	0.10	0.30	0.20
C_{56}	0.855	0.145	0.772	0.785	0.23	2.10	0.25	0.20	0	0.15	0.20	0.20

3.6 Experimental Determination of Compressive Strength

The materials described in section 3.1 were used for the experimental work. Batching of the constituents was done by weight. Cement and fly ash were thoroughly mixed together with a mixture of sand, quarry dust, granite and water. The entire mixes were cast in concrete mouldsof dimensions 150x150x150 mm. The compressive strength tests were carried out in accordance with BS 1881:116 [7]. The concrete cubes were cured under water in a curing tank for a duration of 28days and were thereafter crushed using CompressionTesting Machine. The compressive strength of the cubes was calculated using equation below:

$$Compressive\ strength = \frac{Compressive\ load\ of\ cube\ at\ failure\ (N)}{Cross\ sectional\ area\ of\ cube\ (mm^2)}$$

The progress photograph of the test is shown in Plate 1

PLATE 1:Progress Photograph of Compressive Strength Test

The compressive strength test results of the concrete cubesobtained for the forty-two points of observations are shown in Appendix 2. The mean results are shown in Table 3.

TABLE 3: MEAN EXPERIMENTAL VALUES OF COMPRESSIVE STRENGTH IN N/MM^2 AFTER 28 DAYS WET CURING

Points	Mean Test Result	Points	Mean Test Result	Points	Mean Test Result
Y_1	43.96	Y_{13}	40.84	Y_{26}	38.51
Y_2	42.76	Y_{14}	41.96	Y_{34}	41.00
Y_3	41.11	Y_{15}	41.09	Y_{35}	40.76
Y_4	40.84	Y_{16}	40.76	Y_{36}	38.47
Y_5	38.44	Y_{23}	41.78	Y_{45}	38.49
Y_6	36.56	Y_{24}	41.07	Y_{46}	38.31
Y_{12}	43.11	Y_{25}	40.98	Y_{56}	38.00
C_1	42.84	C_{13}	38.44	C_{26}	40.53
C_2	41.87	C_{14}	40.94	C_{34}	39.33
C_3	41.04	C_{15}	43.11	C_{35}	40.49
C_4	38.58	C_{16}	38.62	C_{36}	39.42
C_5	41.91	C_{23}	42.76	C_{45}	41.02
C_6	39.78	C_{24}	40.96	C_{46}	40.53
C_{12}	41.80	C_{25}	39.51	C_{56}	40.89

3.7 Model for Predictingthe Compressive Strength of Concrete

The mathematical model equation for the optimization of compressive strength concrete containing fly, quarry dust, water and normal aggregates, based on extended Scheffe's of (6,2) factor spacewas obtained by substituting the average experimental values from Table 3 for the first twenty-one points of observations(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{16}, Y_{23}, Y_{24}, Y_{25}, Y_{26}, Y_{34}, Y_{35}, Y_{36}, Y_{45}, Y_{46}, Y_{56}) into Equation (21). The final expression for the model has the form

$Y = 43.96X_1(2X_1$ -1)+42.76 X_2 (2 X_2 -1)+41.11 X_3 (2 X_3 -1)+40.84 X_4 (2 X_4 -1)+38.44 X_5 (2 X_5 -1)+36.56X_6(2X_6 −1)+172.44X_1X_2+171.92X_1X_3+167.84X_1X_4+164.36X_1X_5+163.44X_1X_6 +167.12X_2X_3+164.28X_2X_4+163.92X_2X_5+154.04X_2X_6+164X_3X_4+163.04X_3X_5+153.88X_3X_6+153.36X_4X_5+ 153.24X_4X_6+ 152X_5X_6

The results of predicted compressive strength are shown in Table 4 below

TABLE 4: PREDICTED VALUES OF COMPRESSIVE STRENGTH IN N/MM^2 AFTER 28 DAYS WET CURING

Points	Result	Points	Result	Points	Result
Y_1	43.96	Y_{13}	40.84	Y_{26}	38.51
Y_2	42.76	Y_{14}	41.96	Y_{34}	41.00
Y_3	41.11	Y_{15}	41.09	Y_{35}	40.76
Y_4	40.84	Y_{16}	40.76	Y_{36}	38.47
Y_5	38.44	Y_{23}	41.78	Y_{45}	38.49
Y_6	36.56	Y_{24}	41.07	Y_{46}	38.31
Y_{12}	43.11	Y_{25}	40.98	Y_{56}	38.00
C_1	42.50	C_{13}	39.14	C_{26}	40.08
C_2	41.86	C_{14}	40.44	C_{34}	39.61
C_3	40.20	C_{15}	43.01	C_{35}	40.66
C_4	39.98	C_{16}	39.78	C_{36}	39.90
C_5	41.89	C_{23}	42.31	C_{45}	40.86
C_6	40.59	C_{24}	40.66	C_{46}	40.12
C_{12}	41.89	C_{25}	40.20	C_{56}	40.34

3.8Test for Adequacy of the Model

The test for the adequacy of the model was done using Fischer test at 95% confidence level on the compressive strength at the control points (C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}, C_{18}, C_{19}, C_{20}, C_{21}). In this test two hypotheses were formulated.

Null Hypothesis

There is no significant difference between the laboratory compressive strength of cube test results and model predicted compressive strength results.

Alternative Hypothesis

There is significant difference between the laboratory compressive strength cube test results and model predicted compressive strength results. The hypotheses test was carried out as shown in Table 4.

TABLE 5: FISCHER-STATISTICAL TEST COMPUTATIONS FOR COMPRESSIVE STRENGTH MODEL AFTER 28DAYS WET CURING

Control points	y_e	y_m	$y_e - \overline{y_e}$	$y_m - \overline{y_m}$	$(y_e - \overline{y_e})^2$	$(y_m - \overline{y_m})^2$
C_1	42.84	42.50	2.13	1.74	4.55	3.02
C_2	41.87	41.86	1.16	1.09	1.35	1.20
C_3	41.04	40.20	0.33	-0.56	0.11	0.31
C_4	38.58	39.98	-2.13	-0.78	4.52	0.62

C_5	41.91	41.89	1.20	1.13	1.45	1.27
C_6	39.78	40.59	-0.93	-0.17	0.86	0.03
C_{12}	40.80	41.89	1.09	1.13	1.20	1.27
C_{13}	38.44	39.14	-2.27	-1.62	5.14	2.64
C_{14}	40.94	40.44	0.23	-0.32	0.05	0.10
C_{15}	43.11	43.01	2.40	2.24	5.78	5.03
C_{16}	38.62	39.78	-2..09	-0.98	4.35	0.96
C_{23}	42.76	42.31	2.05	1.55	4.22	2.39
C_{24}	40.96	40.66	0.25	-0.10	0.06	0.01
C_{25}	39.51	40.20	-1.20	-0.56	1.43	0.32
C_{26}	40.53	40.08	-0.18	-0.69	0.03	0.47
C_{34}	39.33	39.61	-1.38	-1.15	1.90	1.32
C_{35}	40.49	40.66	0.25	-0.10	0.06	0.01
C_{36}	39.42	39.90	-1.29	-0.87	1.66	0.75
C_{45}	41.02	40.86	0.31	0.10	0.10	0.01
C_{46}	40.53	40.12	-0.18	-0.64	0.03	0.42
C_{56}	40.89	40.34	0.18	-0.43	0.03	0.18
Sum	$\sum Y_e$= 854.84	$\sum Y_m$= 856.01			$\sum(Y_e - \overline{Y_e})2$= 38.88	$\sum(Y_m - \overline{Y_m})2$=22.34
Mean	$\overline{Y_e}$= 40.71	$\overline{Y_m}$= 40.76				

Here: Y_e is the experimental compressive strength after 28days wet curing and

Y_m is the model predicted compressive strength after 28 days wet curing

$S^2{}_e = \frac{\sum(y_e - \overline{y_e})2}{N-1} = \frac{38.88}{20} = 1.944$

$S^2{}_m = \frac{\sum(y_m - \overline{y_m})2}{N-1} = \frac{22.35}{20} = 1.12$

$F_{calculated} = \frac{S_1^2}{S_2^2}$

where S_1^2 is the greater of S_e^2 and S_m^2, while S_2^2 is the smaller of the two.

Hence, $S_1^2 = S^2{}_e = 1.944$ and $S_2^2 = S^2{}_m = 1.12$

$F_{calculated} = \frac{1.944}{1.12} = 1.74$

The model for compressive strength at 28 days is acceptable at 95% confidence level if:

$\frac{1}{F_\propto(v1,v2)} < \frac{S_1^2}{S_2^2} < F_\propto(v1,v2)$

where significant level, $\propto = 1 - 0.95 = 0.05$; Degree of freedom,

V= N-1= 21-1=20

From standard F- statistic table, $F_\propto$(v1,v2) = 2.12, and $\frac{1}{F_\propto(v1,v2)} = \frac{1}{2.12} = 0.47$

From $\frac{1}{F_\propto(v1,v2)} < \frac{S_1^2}{S_2^2} < F_\propto(20,20)$ which is 0.47 < 1.74< 2.12, this is satisfied

Therefore, the null hypothesis that there is no significant difference between the experimental results and the model expected result is acceptable. This implied that the 28days compressive strength model equation is adequate. for prediction of compressive strength of concrete containing cement, fly ash, sand, quarry dust and water.

3.9 Regression Statistic

Fig.2 below shows the graphical relationship between the experimental and predicted valuesof 28 days compressive strength of the concrete mix considered in this study. The closeness of the data points to the trendline shows that the values of the predicted strength are in agreement with the experimental values. This is evidenced by the value of r^2 of 0.939.

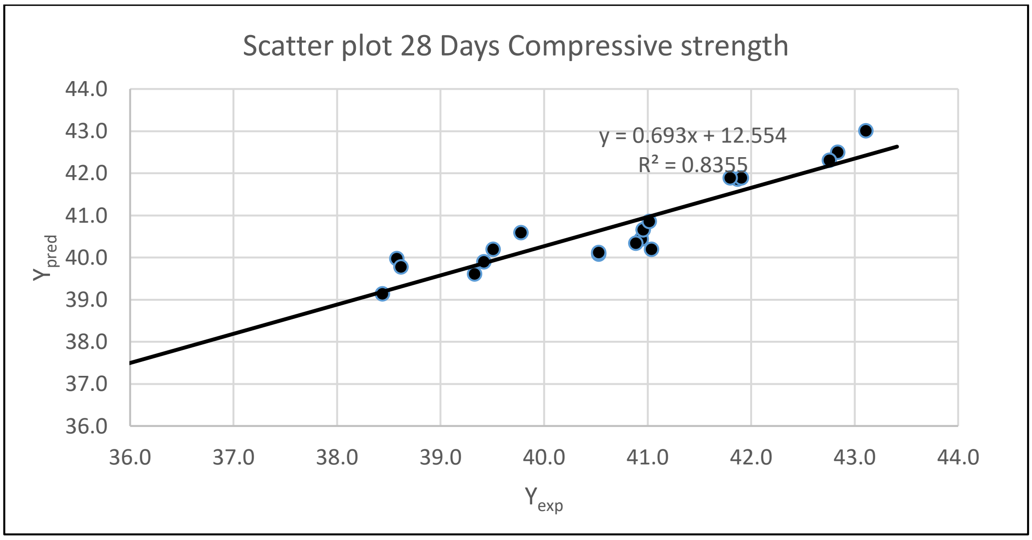

Fig.2: Correlation of Experimental and Predicted 28 days Compressive Strength

4. CONCLUSION

The work considered an extension of Scheffe's optimization techniques from fifth to sixth dimensions, to cover six component mix ratios of concrete containing fly ash and quarry dust and obtained mathematical model for the optimization of the compressive strength. This mathematical model can predict the compressive strength of a six component concretewhen the mix ratios are known. The prediction from the model was tested for 95% accuracy level using Fischer (F) test and regression statistic and found to be adequate with r^2 of 0.84.The maximum strength predicted by the model was 43.01N/mm^2 derived from a mix ratio of 0.91:0.094:0.868:0.138:2.18:0.40. (i.e.,cement : fly ash : sand : quarry dust : granite: water)

REFERENCES

[1] Ephraim M.E. & Rowland-lato E.O. (2015). Compressive strength of concrete made with Quarry Rock Dust and 10mm washed gravel as Aggregate. *American journal of Engineering, Technology and Society*, 2(2), 26-34.

[2] Jamale, A. H., &Kawade, U.R., (2015). Effect of quarry Dust and Fly Ash Mix on strength properties of M40 grade Concrete. *International Journal of Engineering Research and General Science*, 3(3), 2091-2730.

[3] Anyaogu, L. &Ezeh, J.C. (2013). Optimization Compressive strength of fly ash blended Cement Concrete. Using Scheffe's,Simplex Theory, *Academic Research Natural and Applied Sciences* (4)2,2223-9553.

[4] BS 12*:* (1991). *Specification for Portland cement. British Standards Institution*

[5] Scheffe, H. (1958). Experiments with Mixtures, Journal of the Royal Statistical Society. Ser.B., 20, 344 – 360.

[6] Osadebe, N. N. &Ibearuegbulem, O. M. (2009). Application of Osadebe's Alternative Regression Model in Optimizing Compressive Strength of Periwinkle Shell-Granite Concrete, NSE Technical Transaction, 43(1), 47-59.

[7] British Standard (BS) 1881*: Compressive Strength Test of Concrete*

APPENDIX - 1

CONCRETE MIX DESIGN

GRADE OF CONCRETE : C40

Stage	Item	Description	Reference or Calculation			Values	
1		Free w/c Ratio					
	1.1	Characteristic Strength	Specified	N/mm²	after	28	days
				Proportion defective		5	per cent
	1.2	Standard deviation		⁄	N/mm²	8	N/mm²
	1.3	Margin, M		(k=1.64) 1.64x8		13.12	N/mm²
	1.4	Target Mean Strength	C1	fck,cube+M	40+13.12	53.13	N/mm²
					say,	40	N/mm²
	1.5	Cement type	Specified	CEM TYPE OPC			
	1.6	Aggregate type: coarse			Crushed		
		Aggregate type: fine		Uncrushed	(River Sand)		
	1.7	Environment Exposure Class	XC2				
	1.8					BS 5328-1-1997	
		WATER CEMENT RATIO					
			SELECTED	0.40			
2		Water Content					
	2.1	Slump or V-B	S3	Slump Selected		30-75	mm
	2.2	Maximum aggregate size				20	mm
	2.3	Free-water content				206	kg/m³
3		Cement Content	strength condition	206/0.39	=	528	kg/m³
	3.2	Minimum cement content	specified			-	kg/m³
4		Aggregate Content					
	4.1	Relative density of aggregate			2700kg/m³	(tick which is applicable)	
	4.2	Concrete density			2400kg/m³		
	4.3	Density of Cement particles			3150kg/m³		
	4.3	Total aggregate content				1662	kg/m³
5		Fine and Coarse Aggregate Contents					
	5.1	Grading of fine aggregate		Zone 3			
	5.2	Proportion of fine aggregate		32%	say,	32	per cent
	5.3	Fine aggregate content	C5			532	kg/m³
	5.4	Coarse aggregate content				1113	kg/m³

SUMMARY	FINAL RECIPE.			
Mix Proportions	Cement	Water	Fine Aggregate	Coarse Aggregate
	(kg)	*(kg or litre)*	*(kg)*	*(kg)*
Per m³ (to nearest kg)	**528**	**206**	**532**	**1113**

APPENDIX - 2

Experimental Compressive Strength Results after 28 days curing

Points	Test 1	Test 2	Average strength (N/mm^2)	Points	Test 1	Test 2	Average strength (N/mm^2)
Y_1	43.56	44.36	43.96	C_1	42.84	42.67	42.84
Y_2	42.67	42.84	42.76	C_2	41.69	42.04	41.87
Y_3	41.33	41.24	40.84	C_3	41.04	41.04	41.04
Y_4	41.24	40.44	40.84	C_4	38.58	38.76	38.58
Y_5	38.67	38.22	38.44	C_5	41.78	42.04	41.91
Y_6	36.27	36.84	36.56	C_6	39.74	39.82	39.78
Y_{12}	43.56	42.67	43.11	C_{12}	41.38	42.22	41.80
Y_{13}	42.67	43.29	42.98	C_{13}	38.66	38.22	38.44
Y_{14}	41.78	42.13	41.96	C_{14}	41.08	40.08	40.94
Y_{15}	41.29	40.89	41.09	C_{15}	42.67	43.56	43.11
Y_{16}	40.44	41.07	40.76	C_{16}	39.02	38.22	38.62
Y_{23}	42.22	41.33	41.78	C_{23}	42.40	43.11	42.76
Y_{24}	41.24	40.89	41.07	C_{24}	41.42	40.49	40.96
Y_{25}	40.80	41.16	40.98	C_{25}	39.38	39.64	39.51
Y_{26}	37.96	39.07	38.51	C_{26}	41.07	40.00	40.53
Y_{34}	41.07	40.93	41.00	C_{34}	39.02	39.64	39.33
Y_{35}	40.84	40.67	40.76	C_{35}	40.62	40.36	40.49
Y_{36}	38.58	38.36	38.47	C_{36}	39.73	39.11	39.42
Y_{45}	38.58	38.40	38.49	C_{45}	40.98	41.07	41.02
Y_{46}	38.66	37.96	38.31	C_{46}	40.62	40.44	40.53
Y_{56}	37,78	38.22	38.00	C_{56}	40.53	40.34	40.39

APPENDIX - 3

MATLAB CODE

```
%  Concrete Mix Model Prediction
clear
clearall
clc
% Concrete Mix Parameters' Inputs
y1 = input('y1 = ');
y2 = input('y2 = ');
y3 = input('y3 = ');
y4 = input('y4 = ');
y5 = input('y5 = ');
y6 = input('y6 = ');
y12 = input('y12 = ');
y13 = input('y13 = ');
y14 = input('y14 = ');
```

```
y15 = input('y15 = ');
y16 = input('y16 = ');
y23 = input('y23 = ');
y24 = input('y24 = ');
y25 = input('y25 = ');
y26 = input('y26 = ');
y34 = input('y34 = ');
y35 = input('y35 = ');
y36 = input('y36 = ');
y45 = input('y45 = ');
y46 = input('y46 = ');
y56 = input('y56 = ');

% Pseudo Component Input
x1 = input('x1 = ');
x2 = input('x2 = ');
x3 = input('x3 = ');
x4 = input('x4 = ');
x5 = input('x5 = ');
x6 = input('x6 = ');

% Substituted Inputs
N1 = x1*(2*x1-1)*y1 + x2*(2*x2-1)*y2 + x3*(2*x3-1)*y3 + x4*(2*x4-1)*y4;
N2 = x5*(2*x5-1)*y5 + x6*(2*x6-1)*y6 + 4*y12*x1*x2 + 4*y13*x1*x3;
N3 = 4*y14*x1*x4 + 4*y15*x1*x5 + 4*y16*x1*x6 + 4*y23*x2*x3 + 4*y24*x2*x4;
N4 = 4*y25*x2*x5 + 4*y26*x2*x6 + 4*y34*x3*x4 + 4*y35*x3*x5 + 4*y36*x3*x6;
N5 = 4*y45*x4*x5 + 4*y46*x4*x6 + 4*y56*x5*x6;
% Function Response

Y = N1 + N2 + N3 + N4 + N5;
```